Risk in Algorithm Design

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Which is optimal route?
Car Speeds at 77 Mass Ave, Cambridge, MA

Source: Arvind Thiagarajan, Paresh Malalur, CarTel.csail.mit.edu
Optimal $\neq$ short, fast

- Optimal route may differ with start time

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Optimal route with deadline

- \( \min \) Expected Penalty
  
  \[ = 0 \times \text{Prob(on time)} + 1 \times \text{Prob(late)} \]
  
  \[ = \text{Prob(late)} \]

**Remark:** hard to compute

\[ \text{Prob(late)} = \text{Prob}(D_1 + D_2 + \ldots + D_n > t) \]
Grand challenges

Uncertainty

Optimization & Planning

Multiple users
My research

Risk in Algorithm Design

Information (Prediction) markets

Network Auctions

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Talk outline

1. Stochastic route planning with \textit{deadline}

2. Algorithms for Risk-averse problems

3. Multi-user Extensions
Related Work

- Classical shortest path: Dijkstra (1959), …

- Uncertainty & deadline
  - Frank 69 [shortest paths in probabilistic graphs]
  - Loui 83 [linear, exponential & other penalties]
  - Fan, Kalaba, Moore 05 [heuristics]
My contributions on route planning with a deadline

- **Given**: Graph, source, destination
- Edge delays ~ given independent distributions
- **Find** path minimizing
  \[
  \text{Prob(late)} = \text{Prob(path delay} > t)\]

- Distributions:
  - Poisson, gamma(a,b),…
  - Normal
  - Exponential
  - Bernoulli
## My contributions on route planning with a deadline

- **Given**: Graph, source, destination
- Edge delays ~ given independent distributions
- **Find** path minimizing
  \[
  \text{Prob(late)} = \text{Prob(path delay} > t)\]

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Algorithm runtime:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson, gamma(a,b),...</td>
<td>polynomial</td>
</tr>
<tr>
<td>Normal</td>
<td>(n^{\Theta(\log n)})</td>
</tr>
<tr>
<td>Exponential</td>
<td>polynomial, approx.</td>
</tr>
<tr>
<td>Bernoulli</td>
<td>(n^{\text{poly}(\log n)}, \text{approx.})</td>
</tr>
</tbody>
</table>

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Shortest path

- *Optimal substructure (additivity)* property allows for efficient algorithms (Dijkstra, Bellman-Ford, etc)
Planning with deadline: Challenges

- No optimal substructure property (nonadditivity)

Graphical representation of mean and standard deviation for delays in paths A→B, B→C, and A→C.
Planning with deadline: Challenges

- No optimal substructure property (nonadditivity)

PDFs of path delays A→C

- Non-convex objective
Planning with deadline: Analysis

**Insight 1:** \( \text{min } \text{Prob(late)} \)
equivalent to:

\[
\text{minimize} \quad \text{path mean} - t \\
\text{subject to} \quad \sqrt{\text{path var.}}.
\]

**Insight 2:** Visualize on mean-variance plane

\[
\frac{(\text{path mean} - t)^2}{\text{path var.}}
\]

**Insight 3:** Solution is an extreme point!
Theorem 1: Algorithm worst-case running time is $n^{\Theta(\log n)}$.

Algorithm: Enumerate paths minimizing positive combinations of mean, variance. Pick best path.
Theorem 1: Algorithm worst-case running time is $n^{\Theta(\log n)}$.

Algorithm: Enumerate paths minimizing positive combinations of mean, variance.

Pick best path.

Adapted & made practical in: MIT CarTel system for Traffic-aware route planning [S. Lim, H. Balakrishnan, S. Madden, D. Rus, D. Gifford‘08]

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Practical performance

- Algorithm only needs mean, st.devs.
- Heuristic if non-indep, non-Gaussian.
- Running time: better in practice; Boston: 1 sec

Adapted & made practical in: MIT CarTel system for Traffic-aware route planning [S. Lim, H. Balakrishnan, S. Madden, D. Rus, D. Gifford‘08]
Example: 4 pm  MIT to Alewife

Google: “take Mass Av”
21% probability arriving in 20 min

CarTel: >95% probability arriving in 20 min

Google: Mean = 22 mins, stddev = 3 mins
CarTel: Mean = 15 mins (30% better), stddev = 1.5 mins

Source: S. Lim, H. Balakrishnan, cartel.csail.mit.edu
1. Stochastic route planning with *deadline*

2. Algorithms for Risk-averse problems

3. Multi-user Extensions
Problem formulation

• Given a combinatorial problem
  Feasible set $F$ in $\{0,1\}^n$
  Random independent costs with
    means $\mu = (\mu_1, \ldots, \mu_n)$, variances $\nu = (\nu_1, \ldots, \nu_n)$
  minimize risk-averse objective($x$)
  subject to $x$ in $F$.

• We solve this via oracle calls to algorithm for deterministic problem
Risk-averse optimization framework

We can solve:

- $\text{Min } \Pr(\text{cost} > \text{Budget})$
- $\text{Min } \text{Budget, such that } \Pr(\text{cost} \leq \text{Budget}) \geq p$
- $\text{Min mean + } k \times \text{st.dev}$

over arbitrary feasible set

Risk-averse optimization: Exact Algorithm

Exact Algorithm:
Enumerate feasible solutions minimizing positive combinations of mean, variance.
Pick best one.

Theorem 1: subexponential for shortest paths
Corollary 2: poly-time for MST and matroids;
Risk-averse optimization: Main result

Theorem 3: There is an \((1+\varepsilon)\)-approximation algorithm for stochastic problem below that uses poly( |input|, \(1/\varepsilon\)) queries to exact algorithm for deterministic problem.

\[
\min \text{ risk-averse objective}(x) \\
\text{s.t. } x \in F
\]

\begin{itemize}
  \item \(\min \text{ mean}(x) + k \sqrt{\text{variance}(x)}\)
  \item \(\min \Pr( \text{Cost}(x) \geq \text{Budget}) \) *
  \item \(\min B, \text{such that } \Pr( \text{Cost}(x) \geq B) \leq p \) *
\end{itemize}

* Exact objective for Normal dist; Upper Bound for arbitrary dist.

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Risk-averse optimization: Main result

Theorem 4: There is an \((1+\varepsilon)\delta\)-approximation** algorithm for stochastic problem below that uses \(\text{poly}(|\text{input}|, 1/\varepsilon)\) queries to \(\delta\)-approx. algorithm for deterministic problem.

\[
\begin{align*}
\min \text{ risk-averse objective}(x) \\
\text{s.t. } x \in F
\end{align*}
\]

- \(\min \text{ mean}(x) + k \sqrt{\text{variance}(x)}\)
- \(\min \Pr(\text{ Cost}(x) \geq \text{Budget})\) *
- \(\min B, \text{such that } \Pr(\text{ Cost}(x) \geq B) \leq p\) *

* Exact objective for Normal dist; Upper Bound for arbitrary dist.

** More complicated approx. factor for \(\min \text{ Prob. Objective.}\)
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Cost of Congestion

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Extensions to multiple users

Summary & Future

• **New algorithms** for risk-averse users

• **Strategic challenges** from multiple users

• **New theory needed** in information age

Future: Unified foundation of algorithms merging stochastic, dynamic & economic analysis