Operational amplifiers

Building blocks of servos
Operational amplifiers (Op-amps)

Want perfect amplifier
- Infinite gain
- Infinite input impedance
  - will not load down source
- Zero output impedance
  - will drive anything

Have operational amplifiers
- Gain ~ $10^6$
- Input impedance ~ 100 MΩ
- Output impedance ~ 100 Ω

Problems
- Gain too high
  - slightest input noise causes max output
- Other problems to be discussed later

Solutions
- Use feedback
- Gain depends only on resistance: $R_f / R_{in}$
  - can control precisely

Op amp with feedback

\[ V_1 = (V_{out} - V_{in}) \frac{R_{in}}{R_{in} + R_f} + V_{in} \]

\[ V_{out} = -A \ V_1 \]

\[ V_{out} = -V_{in} \left( \frac{R_f}{R_{in}} \right) / \left[ 1 + \left( 1 + \frac{R_f}{R_{in}} \right) / A \right] \]

Small gain:
\[ V_{out} = -A \ V_{in} \frac{R_f}{(R_f + R_{in})} \sim \text{divider} \]

Large gain:
\[ V_{out} = -V_{in} \left( \frac{R_f}{R_{in}} \right) \]

Note: \[ V_1 = -V_{out} / A \sim 0 \]
Differential amplifier

- Op amp output actually depends on voltage difference at two inputs
- \[ V_{out} = -(V_{in1} - V_{in2}) \left( \frac{R_f}{R_{in}} \right) \]
- Insensitivity to common voltage at both inputs = CMRR
- Real op amps have problems with unbalanced input impedance
  - Feedback resistor creates imbalance

Solution:
- Add input resistor and potentiometer
- Op amp with built-in resistors = instrumentation amp.

**Ideal op-amp circuit**

**Non-ideal op-amp**

Resistors to compensate for non-ideal op-amp ~ \( R_f \)
Integrators

- Put capacitor in op amp feedback path
  - \( V_{\text{out}} = -V_{\text{in}} \left( \frac{Z_f}{R_{\text{in}}} \right) = -V_{\text{in}} / (2 \pi j f \ C \ R_{\text{in}}) \)
- Similar to low pass filter in high frequency limit
  - except applies to low frequencies also
  - can show large gain near dc
- Recall \( V_1 \sim 0 \) forces \( I_{\text{in}} = -I_{\text{feedback}} \)
  - charge on capacitor is integral of \( I_f \)
  - since \( V_{\text{out}} = Q/C_f \), \( V_{\text{out}} \) is integral of \( I_{\text{in}} \)
- **Result is integrator**
  - integration speed \( \sim 1 / R_{\text{in}} C_f \)

\[ \begin{align*}
\text{Integrator} & \\
V_{\text{in}} & \rightarrow R_{\text{in}} \rightarrow V_1 \\
I_{\text{in}} & \rightarrow -I_f \\
C_f & \rightarrow V_{\text{out}}
\end{align*} \]

**Gain response**
- Single-pole rolloff
  - \( 6 \text{ dB/octave} \)
  - \( 10 \text{ dB/decade} \ \text{RC} \)
- Unity gain at \( f = 1 / 2 \pi RC \)
- \( >60\text{dB} \)

**Phase response**
- 60 dB
- 0 degrees
- -90 degrees
Shunted integrator

- Limit dc gain
- Advantages:
  - dc input voltage no longer saturates op amp output
  - prevents servo runaway
- Dis-advantages
  - long term errors not well corrected by servo

\[
\log(V_{out}/V_{in}) = \log(f)
\]

Gain response
Max gain = \( R_f/R_{in} \) at \( f < 1/2 \pi R_f C_f \)

Unity gain at \( f = 1/2 \pi R_{in} C_f \)

Phase response
Phase shift
-90 degrees
0 degrees
Real op amp

Op amp without feedback
- Acts like shunted integrator
Stability condition:
- unity gain freq. before second pole
- otherwise feedback becomes positive
  - oscillation

Real op amp

![Real op amp diagram](image)

Gain response
- Max gain
- Single pole 6 dB
- Double pole 12 dB

Phase response
- 0 degrees
- -90 degrees
- -180 degrees

\[
\log(V_{\text{out}}/V_{\text{in}}) \quad \log(f)
\]
Integrator with lead

High frequency gain has minimum value

Purpose:
- Provides phase lead
- Can compensate for pole in servo

Alternate circuits for lead
- Capacitor at input, inductor in feedback
- Overall positive phase
  - analog to faster than light propagation
  - output anticipates input

\[
\begin{align*}
\text{Integrator with lead} \\
V_{\text{in}} & \quad \text{R}_{\text{in}} \\
\text{I}_{\text{in}} & \quad V_{1} \\
\text{V}_{1} & \quad \text{R}_{f} \\
\text{C}_{f} & \quad \text{I}_{f} \\
\text{V}_{\text{out}} & 
\end{align*}
\]

Gain response

\[
\log\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)
\]

Phase response

\[
\text{Phase shift}
\]

- 0 degrees
- -90 degrees

Single pole

6 dB

Min gain
Summing amplifier

• High gain forces $V_1 \sim 0$
• Feedback current must cancel input current
• Generalize to multiple inputs
  - $V_{out} = -R_{feedback} \sum I_{in} = -(R_f / R_{in}) \sum V_{in}$
  - Works because $V_1 \sim 0$
• Summing amplifier
  - Op amp input is summing junction
  - Useful for combining multiple inputs
Drift-compensated integrator

Real op amps have leakage current

- Can saturate integrator
- Compensate with dc current to summing junction

Drift-compensated integrator

![Diagram of Drift-compensated integrator](image)

- $V_{in}$
- $R_{in}$
- $I_{in}$
- $R_c$
- $I_c$
- $V_1$
- $I_f$
- $C_f$
- $V_{out}$

Integrator leakage compensation
Trans-impedance amplifiers

- Input is current source
  - model as voltage source with high impedance
  - \( I_{\text{in}} = \frac{V_{\text{source}}}{Z_{\text{source}}} \)
  - \( V_{\text{out}} = -Z_{\text{feedback}} \frac{V_{\text{source}}}{Z_{\text{source}}} = -I_{\text{in}} Z_{\text{feedback}} \)

- Trans-impedance amplifier
  - current in, voltage out
  - gain expressed in Ohms
Photodiode amplifier

- Photodiode like current source but with capacitor
- Input capacitor causes op amp gain to diverge at high freq.
  - Amplifies high freq noise
  - Oscillation
- Solution:
  - Shunt capacitor in feedback

Gain response

\[
\log \left( \frac{V_{\text{out}}}{I_{\text{in}}} \right) \quad \log(f)
\]

Unshunted

Shunted
Integrator design tips

Shunting switch with small value resistor
- Discharges capacitor
  - initialize integrator/ servo
  - simplify servo testing allow rest of circuit

Resistors on power supply rails
- Limits current to saturated op amp
  - prevents burn out

Capacitors on supply rails
- Reduces noise

Note on capacitors
- High values
  - electrolytic, polar
  - leakage resistance
- Use low value in parallel