

Probabilistic Modeling of Leach Protocol and Computing Sensor Energy Consumption Rate in Sensor Networks

Dezhen Song

CS Department, Texas A&M University

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Email: dzsong@cs.tamu.edu

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Abstract

One important performance index of routing mechanisms in sensor networks is sensor lifespan. In our settings, sensors are energy-constrained by batteries and located far away from the access point. A sensor dies once its battery dies. Since the radio transmission consumes a lot of energy, researchers proposed several routing mechanisms to save energy to prolong the sensor lifespan. So far, the comparison of different routing mechanisms are based on simulation and little analytical results were available. In this project, we use graph and stochastic modeling techniques to compare the sensor lifespan among different routing mechanisms including direct transmission, minimum energy transmission, static clustering, and dynamic clustering.

1 Introduction

Sensor networks become more and more popular as cost of sensor gets cheaper and cheaper. The sensor network is a wireless network formed by a group of sensors deployed in same region, which can be used to measure air pressure, temperature, acceleration, etc. Sensors transmit signals via radio signal. Since sensors are now small and cheap, they can be deployed in large scale. They become more and more important for applications like security, traffic monitoring, agriculture, war field, etc.

Most of those sensors are powered by batteries. The lifespan of a energy-constrained sensor is determined by how fast the sensor consumes energy. Sensors use energy to run circuitry and send radio signals. The later is usually a function of distance and takes a large portion of the energy. Researchers are now developing new routing mechanisms for sensor networks to save energy and pro-

long the sensor lifespan. Four primary routing mechanisms are direct transmission, minimum energy transmission, static clustering, and dynamic clustering.

Sensor lifespan is an important performance index for comparison of different routing mechanisms. So far, the comparison between routing mechanisms is based on simulation results. To author's knowledge, little analytical results have been available. In this project, we will use graph theory and stochastic modelling techniques to assess the sensor lifespan quantitatively.

2 Related work

In [1], Heinzelman, Chandrakasan, and Balakrishnan propose Low-Energy Adaptive Clustering Hierarchy (LEACH), which is an energy-efficient communication protocol for wireless microsensor networks. The application scenario is,

- The base station is fixed and located far from the sensors.
- All nodes in the network are homogeneous and energy-constrained.

LEACH is a dynamic clustering method. In this method, time is partitioned into fixed intervals with equal length. At the beginning of each interval, each sensor becomes a cluster head with some predefined probability. The cluster heads then broadcast messages to their neighbors. Other sensors receive messages and join a cluster by choosing the cluster head with the strongest signal. During the interval, cluster members send information to their cluster head. The cluster heads aggregate the information, compress the information, and route the information to the remote access point. Once the interval ends, the whole clustering process restarts. Hence, the clusters and cluster heads are not fixed. Since the cluster heads consume more energy than cluster members in radio transmission, the rotation of cluster heads makes energy consumption more evenly across all sensors in the network. Therefore, the sensor network can last longer. In [1], Heinzelman et al. compare LEACH with direct transmission, minimum energy transmission, and static clustering. The simulation results show that the LEACH can extend the sensor network life up to eight times longer than its closest competitors.

In this project, we follow the same assumptions in [1] and use graph theory and stochastic modelling techniques to explicitly compute the sensor life time for the four routing mechanisms.

[I WILL ADD MORE RELATED WORK IN PAPER VERSION. FOR PROJECT REPORT, I DO NOT HAVE TIME TO DO SO.]

3 Problem definition

In this section, we first introduce inputs and assumptions of the sensor network system. We try to follow the assumptions in [1] so that we can compare the simulation result with our analytical model later.

3.1 Inputs and assumptions

Sensors are identical and energy constrained. Since we are interested in studying the energy efficiency, we assume that sensor lifespan only depends on energy. A sensor dies only if its batter dies.

3.1.1 Energy models

As mentioned in [1], a sensor consumes energy in two ways: the energy used to run circuitry and the energy used to send radio transmission. The energy used to run circuitry is proportional to the number of bits in the message. Say that the message length is k bit, define E_{elec} be the energy per bit. Then the energy used to run circuitry is $E_{elec}k$. This energy is the same for receiver and transmitter. The energy for transmitter to send k bits over distance d is $\epsilon_{amp}kd^2$, where ϵ_{amp} is the energy constant for the radio transmission. Therefore, the total energy for a receiver to handle a k -bit message is,

$$E_{Rx}(k) = E_{elec}k. \quad (1)$$

The total energy for a transmitter to send a k -bit message over distance d is,

$$E_{Tx}(k, d) = E_{elec}k + \epsilon_{amp}kd^2. \quad (2)$$

We name this energy model as H-model because it was proposed by Heinzelman et al. We will follow this energy model in our modelling. As we will show later, our modelling methods can be applied to different variations of energy models.

3.1.2 Sensor network settings

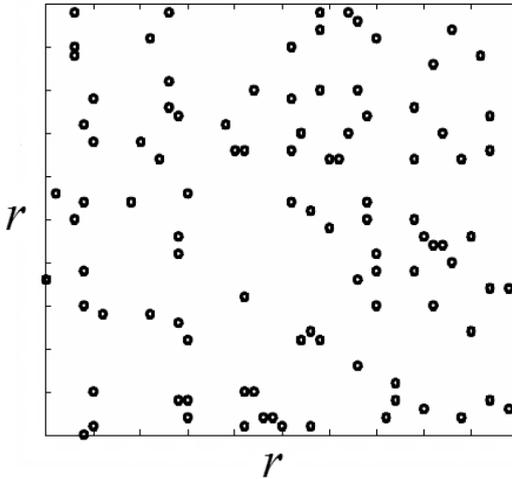


Figure 1: 100 sensors are uniformly randomly distributed in a $r \times r$ square.

As shown in figure 1, sensors are uniformly distributed in a $r \times r$ square. All sensors are identical and constrained with same amount of energy. There are 100 sensors in total in the sensor network. The access point/base station is located far from the network.

3.2 Problem formulation

We formulate our problem as: given the energy model, sensor network settings, and routing mechanism, find the sensor lifespan. Since sensor lifespan is related to battery capacity, we are more interested in study long run rate of energy consumption of a given sensor, which fully depends on our inputs. If there are random factors in the routing mechanism, then we use expected rate of energy consumption instead. Therefore, the alternative problem definition is,

Inputs Energy model, sensor network settings, and routing mechanism, The sensor location (Sometimes the sensor is referred as our sensor in the rest of the paper.)

Output Quantitative model of (Expected) rate of energy consumption of the sensor.

Since we have more than one type of routing mechanisms, we deal with them individually in next section. In direct transmissions, minimum-transmission-energy, and static clustering model, we assume that we know the exact position of the 100 sensors. In the dynamic clustering model, we will assume that we only know our sensor's location and other sensors are randomly distributed over the squared region.

4 Energy consumption quantitative models

In this section, we establish quantitative models for different routing mechanism. We start from the direct transmission, which is the simplest model.

4.1 Modelling direct transmission

Direct transmission means that each sensor directly send its own message to the base station. Let's define,

- d_b be the distance between the sensor and the access point,
- t be a fixed time interval,
- k be number of bits send over the fixed period t .

Plug those variables into equation 2, we know that the energy consumption rate γ is,

$$\gamma = \frac{E_{elec}k + \epsilon_{amp}kd_b^2}{t}. \quad (3)$$

4.2 Modelling minimum-transmission-energy

Recall that the energy used to transmit k -bit message over distance d is $\epsilon_{amp}kd^2$. Minimum transmission energy select a route that minimize energy on radio transmission. In other words, the energy spent on running the circuitry is ignored. Figure 1 compares the direct transmission and minimum transmission

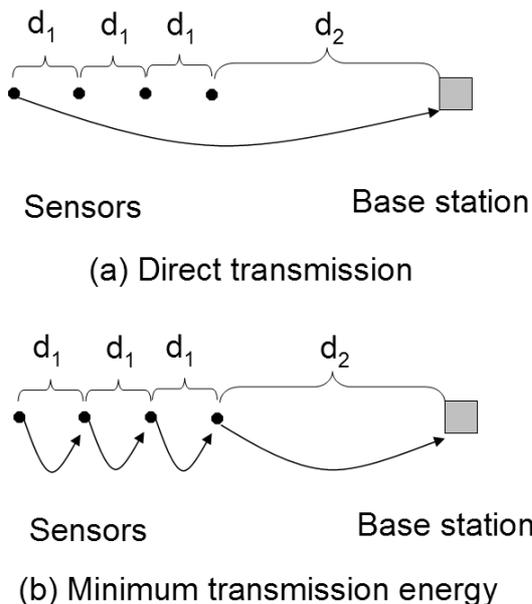


Figure 2: *Comparison between direct transmission and minimum transmission energy hopping.*

energy hopping using a case that there are only 4 collinear sensors and one base station. In figure 2(a), the radio transmission energy used to transmit message from the leftmost sensor to the base station is $\epsilon_{amp}k(3d_1 + d_2)^2$. In figure 2(b), the radio transmission energy used to transmit message from the leftmost sensor to the base station is $\epsilon_{amp}k(3d_1^2 + d_2^2)$, which is a lot less than the direct transmission.

Figure 2 uses a very simple settings. To apply minimum transmission hopping method to sensor network shown in figure 1 is not as straightforward. There are two problems. One is how to find a minimum transmission energy hoping route and the other is how to keep track of energy speed on transmitting and routing. Let's define,

- n_r be number of messages routing through the sensor,
- d_r be distance between the sensor and the next neighboring sensor in route, which receives the messages from our sensor. We will show how to find it shortly.

Plug them into the energy model, we know that the energy consumption rate for the sensor is,

$$\gamma = \frac{(2n_r + 1)E_{elec}k + (n_r + 1)\epsilon_{amp}kd_r^2}{t}. \quad (4)$$

The reason for $2n_r + 1$ is that for each routed message, we need to receive and transmit. The circuitry is fired up twice. There are n_r messages to route through the sensor and the sensor has its own message. The remaining problem is how to compute n_r and d_r for a given set of sensor allocations. We can construct a graph $G(V, E)$, where V the set of vertices and E is the set of edges. Set V consists of all sensors and the access point/base station. There are 101 vertices in our case. We number the access point as v_0 and sensors as v_i , $1 \leq i \leq 100$. An edge e_{ij} is the squared Euclidean distance between vertex i and j . Set E is the collection of edges. If we set v_0 as the destination and solve the shortest path problem using Dijkstra's algorithm. The result will give us the minimum transmission energy hopping routing results, which include n_r and d_r .

Define (x_i, y_i) be coordinate of the vertex v_i . If we modify the edge length in $G(V, E)$ to be $e_{ij} = 2E_{elec}k + \epsilon_{amp}k((x_i - x_j)^2 + (y_i - y_j)^2)$, then the minimum transmission energy hopping becomes real minimum energy hopping. This modification can give results for this new routing mechanism.

4.3 Modelling static clustering

In static clustering routing mechanism, some sensors act as cluster heads and others act as cluster members. Each cluster member chooses the nearest cluster head as its cluster head. During the transmission, only cluster heads talk to the access point. Cluster members send their messages to cluster heads and the cluster heads route their messages to the access point. Therefore, for the given sensor, they are two cases: cluster member or cluster head.

4.3.1 Cluster member

As a cluster member, all the sensor needs to do is to send its message to its cluster head. Define d_c be the distance between the sensor and its cluster head, then the energy consumption rate is,

$$\gamma = \frac{E_{elec}k + \epsilon_{amp}kd_c^2}{t}. \quad (5)$$

4.3.2 Cluster head

As cluster head, the sensor needs to route all member messages and send its own message to the access point. Define,

- n_c be number of cluster members,
- d_a be the distance between the sensor and the access point.

then the energy consumption rate for the sensor is,

$$\gamma = \frac{(2n_c + 1)E_{elec}k + (n_c + 1)\epsilon_{amp}kd_a^2}{t}. \quad (6)$$

The remaining problem is how to obtain d_c and n_c . This can be computed using algorithm for Voronoi Diagram based on squared Euclidean distance.

4.4 Modelling dynamic clustering (LEACH)

As reviewed in related work section, we know that LEACH is a dynamic clustering based routing mechanism. Due to the random facts, we can not compute the sensor energy consumption rate exactly. However, we can compute the expected energy consumption rate $E(\gamma)$. Let's begin with the review of some stochastic processes.

4.4.1 Renewal reward process

A counting process $N(t)$ counts number of events up to time t . A renewal process is a counting process such that its inter-arrival time are identically independent distributed (*iid*) random variables. A renewal reward process is a renewal process such that there exists *iid* reward for each inter-arrival time. Let's define,

- X_i be inter-arrival time, $X_i, i = 0, 1, 2, \dots$,
- R_i be reward for the inter-arrival time X_i ,
- $R(t) = \sum_{i=1}^{N(t)} R_i$ be total reward earned up to time t .

According to renewal reward theorem, we know that,

$$\lim_{t \rightarrow \infty} \frac{R(t)}{t} = \frac{E(R)}{E(X)}. \quad (7)$$

This means that the long run rate of reward is equal to the ratio between the expected reward and the expected inter-arrival time in a single interval. Proof of the theorem is based on Strong Law of Large Numbers, which is beyond the scope of the report. In stochastic process, the interval time is called cycle.

The power of the renewal reward process lies on the fact that it does not depend on any distribution. We can apply the renewal reward theorem to our problem. If we treat the energy consumed by the sensor as the reward, then the long run rate of reward is essentially the long run rate of energy consumption, which is what we are looking for. The difficult problem is how to define cycle such that the reward and the cycle are both *iid* random variables. We define a cycle be the number of intervals between two consecutive moments that the sensor becomes cluster head. The cycle/inter-arrival time is an integer number indicating number of intervals. Recall that the interval is a period that clusters and cluster heads are regenerated. Now our problem becomes how to compute $E(R)$ and $E(X)$.

4.4.2 Computing $E(X)$

At the beginning of each time interval t_d , clusters and cluster heads are regenerated. At first, each sensor i , $1 \leq i \leq 100$ generates a random number between 0 and 1 and compares it to a predefined threshold $T(i)$. If the random number is less than $T(i)$, the sensor becomes cluster head. Otherwise, the sensor acts as a cluster member. Define

- P be the desired percentage of cluster heads,
- $n = 1/P$,
- m be the current round in terms of intervals,
- G be set of sensors that have not been cluster-heads in the last n rounds.

According to [1], the threshold for sensor i is,

$$T(i) = \begin{cases} \frac{P}{1-P(m \bmod n)} & \text{if sensor } i \in G, \\ 0 & \text{otherwise} \end{cases}$$

After look into this cluster head generation method further, we find that it embeds a Markov chain if we define state variable S be number of intervals that the sensor has not been a cluster head yet. The state $S = 0$ means the the sensor is currently a cluster head. We can draw the transition diagram of this Markov chain as the following,

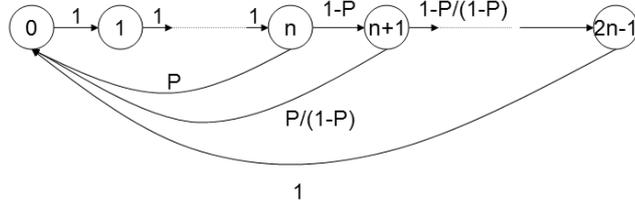


Figure 3: A Markov chain model of sensor states.

All states in this Markov chain communicate with each other. We can compute the limiting probabilities for each state, which is also the long run probability the chain stays in the state. Define π_0 be limiting probability for the chain stays in state 0. Note that this is also the probability that the sensor becomes a cluster head. Therefore, the expected cycle length has to be,

$$E(X) = \frac{1}{\pi_0} \quad (8)$$

4.4.3 Computing $E(R)$

During a cycle, the sensor consumes energy differently when its role in the cluster is different. Define,

- M be the amount of energy consumed in a single interval as a cluster member,
- H be the amount of energy consumed in a single interval as a cluster head.

A cycle consists of X intervals. Among them, the sensor is a cluster member for $X - 1$ intervals and a cluster head for 1 interval. Therefore, the amount energy consumed in a single cycle is,

$$E(R) = (E(X) - 1)E(M) + E(H). \quad (9)$$

The remaining problem is $E(M)$ and $E(H)$. We start with $E(M)$. We can use the result from static clustering routing mechanism, we know that if the distance between the sensor the nearest cluster head is D_c , then according to equation 5, we have the conditional expectation,

$$E(M|D_c) = E_{elec}k + \epsilon_{amp}kD_c^2 \quad (10)$$

Define $f(a)$ be the distribution function of D_c . The unconditional expectation depends on $f(a)$,

$$E(M) = \int_0^\infty E(M|D_c = a)f(a)da = E_{elec}k + \epsilon_{amp}k \int_0^\infty a^2 f(a)da \quad (11)$$

Define $P\{D_c > a\}$ be the probability that the distance between the sensor and the nearest cluster head is greater than a . Then we know,

$$f(a) = 1 - \frac{d}{da}P\{D_c > a\}. \quad (12)$$

Define N_c be number of clusters. We know that $P\{D_c > a\}$ depends on N_c . Given there are N_c cluster heads, $P\{D_c > a|N_c\}$ is equivalent to the probability that the distance between the sensor and any cluster head is greater than a . Define D_1 the distance between the sensor and one of the cluster head. Then we have,

$$P\{D_c > a|N_c\} = (P\{D_1 > a\})^{N_c}. \quad (13)$$

Since each sensor can become cluster head with probability π_0 , N_c has to be a binomially distributed random variable,

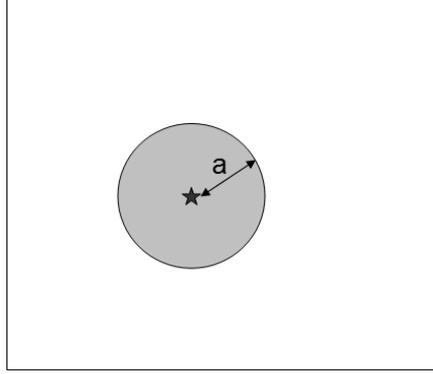
$$P\{N_c = n\} = \binom{99}{n} \pi_0^n (1 - \pi_0)^{99-n} \quad (14)$$

Therefore,

$$P\{D_c > a\} = \sum_{n=1}^{99} P\{D_c > a|N_c = n\}P\{N_c = n\} \quad (15)$$

$$= \sum_{n=1}^{99} (P\{D_1 > a\})^n \binom{99}{n} \pi_0^n (1 - \pi_0)^{99-n} \quad (16)$$

Then the remain the problem left for computing $E(M)$ is how to obtain $P\{D_1 > a\}$. The meaning of $P\{D_1 > a\}$ is the probability that the distance between the sensor and a cluster head is greater than a . Since a cluster head is a sensor, which is uniformly randomly distributed in the space. If we pick up a point which is uniformly randomly distributed in the squared region, $P\{D_1 > a\}$ is the probability that the distance between the point and the sensor is greater than a . Figure 5 illustrates how to compute the $P\{D_1 > a\}$,



★ The sensor

Figure 4: $P\{D_1 > a\}$ illustration

$$P\{D_1 > a\} = 1 - \frac{Area(Circle \cap Square)}{Area(Square)} \quad (17)$$

Using equation 10 ~ 17, we can compute $E(M)$. Now we concentrate on how to compute $E(H)$. Since the cluster head aggregates the sensor data and compresses them before send them to the access point, the actually message length sent by the cluster head is less than the summation of all messages. Define γ the the compression ratio. Given there are N_m cluster members, the energy for the cluster head should be,

$$E(H|N_m) = (2N_m + 1)kE_{elec} + (N_m + 1)\epsilon_{amp}k\gamma d_a^2. \quad (18)$$

The the conditional expectation should be,

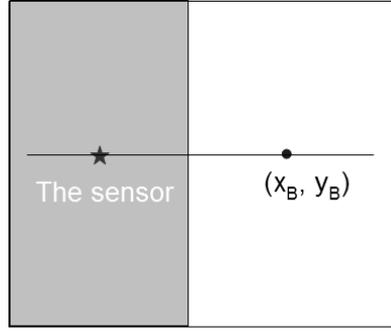
$$E(H) = \sum_{i=0}^{99} E(H|N_m = i)P\{N_m = i\}. \quad (19)$$

To get the unconditional expectation of H , we need to know the probability distribution of N_m , which is related to another random variables, number of clusters N_c other than the sensor. As shown in equation 14, the number of

clusters should be a binomial variable. We condition on number of clusters $P\{N_m = i|N_c\}$. If we can compute $P\{N_m = i|N_c\}$, then it follows

$$P\{N_m = i\} = \sum_0^{99} P\{N_m = i|N_c = n\}P\{N_c = n\}. \quad (20)$$

Knowing there are n clusters other than our sensor is a very helpful information. A randomly located sensor enters the system and try to find the closest cluster head. In order to be the closest cluster, our sensor has to win the distance contest over all other n cluster heads. This can be viewed as n independent contests. They are independent because the probability of winning the contest depends on the locations of other cluster heads, which are independent. In fact, n contests are independent and symmetric. Define event A be case that our sensor win the contest over cluster head B when a member sensor enters the system. If we condition on the location of cluster head B is (x_B, y_B) , then we can compute the probability event A happens: $P\{A|(x_B, y_B)\}$. Let us draw a



★ The sensor • Cluster head B

Figure 5: $P\{A|(x_B, y_B)\}$ illustration

line segment between sensor B and the sensor. We can find the middle point of the line segment and draw a line perpendicular to the line segment. The line divides the square into two regions. The region contains our sensor is the grayed region. It is clear that any sensor located in grayed region chooses the sensor as their cluster head, which means,

$$P\{A|(x_B, y_B)\} = \frac{Area(Grayed Region)}{Area(Square)} \quad (21)$$

The unconditional probability can be computed,

$$P\{A\} = \iint_{square} \frac{Area(Grayed Region)}{Area(Square)} \frac{1}{r^2} dx dy \quad (22)$$

$$= \iint_{square} \frac{Area(Grayed Region)}{r^4} dx dy \quad (23)$$

With $P\{A\}$, we are now ready to compute the conditional probability $P\{N_m = i | N_c = n\}$, which is the probability that our cluster has i member sensors given that there are n other cluster heads. If there are n other cluster heads, there are $99 - n$ member sensors. When each member cluster enters the system, it will choose our cluster head as its closest cluster head with probability $(P\{A\})^n$ since our cluster head has to win n independent vicinity contests. Therefore, the probability that our cluster has i members under those settings should be,

$$P\{N_m = i | N_c = n\} = \binom{99 - n}{i} ((P\{A\})^n)^i (1 - P\{A\})^{99 - n - i} \quad (24)$$

Combine equation 18 ~ 24, we can compute the $E(H)$. Hence, we can get $E(R)$ using equation 9.

5 Conclusion and future work

Quantitative assessment of sensor lifespan in a sensor network can help us to improve the routing mechanism. The modelling process can point out what affect the sensor life and what is the weak point of the routing protocol, which could be difficult if using simulation-based approach.

In the future, we will provide numerical results for the four routing mechanism and compare them to simulation results. One interesting problem is to design or modifying existing routing strategies to synchronize the sensor lifespan in a sensor network. This involves two efforts including how to make expected sensor lifespan to be the same and how to reduce the variance of the sensor lifespan. The synchronization of lifespan will significantly reduce the maintenance cost of a sensor network.

References

- [1] W. R. Heinzelman, A. Chandrakasan, and H. Balakrishnan. Energy-efficient communication protocol for wireless microsensor networks. In *Proceedings of the Hawaii International Conference on System Sciences, Maui, Hawaii*, Jan 2000.